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SIMPLIFICATION

Formula
1. \((a + b)^2 = a^2 + 2ab + b^2\)
2. \((a - b)^2 = a^2 - 2ab + b^2\)
3. \((a + b)(a - b) = a^2 - b^2\)
4. \((a + b)^2 = (a - b)^2 + 4ab\)
5. \((a + b)^2 + (a - b)^2 = 2(a^2 + b^2)\)
6. \((a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)\)
7. \((a + b)^3 = a^3 + b^3 + 3ab(a + b)\)
8. \((a - b)^3 = a^3 - b^3 - 3ab(a - b)\)
9. \(a^3 + b^3 + c^3 - 3abc = (a + b + c)\)
   \((a^2 + b^2 + c^2 - ab - ac - bc)\)
   if \(a + b + c = 0\), then \(a^1 + b^1 + c^1 = 3abc\)

Square of any number
Square of a number ⇒ "multiply a number by itself".
In this section we shall learn some powerful methods to compute the square of any number within few seconds. First we have to learn few points which are related to this topic.

Deviation
Number is how much more or less than a number (let X). Here X is a number which is multiple of 50.

For example
let \(X = 56\),
Deviation is \((1 \times 6)\) because 56 is 6 more than 50.
let \(X = 46\)
Deviation is \((1 \times 4)\) because 46 is 4 less than 50
let \(X = 109\)
Deviation is 9 because 109 is 9 more than 100.
Let \(X = 152\)
Deviation is \((3 \times 2)\) because 152 is 2 more than 150.

Base
All numbers which are multiple of 50, (except those numbers which have integer power of 10 for eg, \(10^1 = 10\), \(10^2 = 100\), \(10^3 = 1000\)) for eg, 50, 150, 200, 250 etc. Eliminate zero & take square of remaining number is called base. For example base of 50.
⇒ Eliminate zero & take square of 5 ⇒ \(5^2 = 25\).
Base of 150 ⇒ \(15^2 = 225\).

For 10, 100, 1000 we take number itself as a base.
For example 92 is near to 100, so for finding the square of 92 we take 92 as a base.

1. Square of a number which is near 50
let we have to find the square of \(N\). We use the following formula for it
\[N^2 = (25 \pm \text{deviation})^2\]
\[\Rightarrow 2\text{-digit}\]

First find the deviation if number is more than 50 then add the deviation in 25, if number is less than 50 then subtract the deviation from 25. Second write the square of deviation right side of it, remember it should always be in 2 digits. If it is in one digit put zero before it, if it is in 3 or 4 digit transfer the carry left side.

- \(54^2 \Rightarrow 25 + 4\)
- \(\frac{29}{4^2}\)
- \(54^2 = 2916\)

**Step 1:** Deviation is 4, add 4 in 25 because 54 is more than 50.

- \(58^2 \Rightarrow 25 + 8\)
- \(58^2 = 3364\)

**Step 2:** Write square of deviation right side of it.

- \(48^2 \Rightarrow 25 - 2\)
- \(48^2 = 2304\)

- \(61^2 \Rightarrow 25 + 11\)
- \(61^2 = 3721\)

\[36/121\]

- \(61^2 \Rightarrow 25 + 16\)
- \(41/256\)

**Step 3:** 61 is 11 more than 50 so deviation is 11.

Her deviation is 16, square of deviation is 256.

- \(38^2 \Rightarrow 25 - 12\)
- \(38^2 = 1444\)

\[\because 38 \text{ is } 12 \text{ less than } 50, \text{ square of } 12 \text{ is } 144\]

- \(42^2 \Rightarrow 25 - 8\)
- \(42^2 = 1764\)
2. Square of a number which is near 100

Let we have to find the square of 91. If we use the formula near 50, then it is a bit complicated, but we can find it easily by the rule of square near to 100.

Use the following formula for finding the square near 100:

\[ N^2 = (N \pm \text{deviation})^2 \]

- \[ 91^2 \Rightarrow 91 \]
  \[ \begin{array}{c}
  \text{91 is 9 less than 100 so here deviation is 9. Take 91 as a base subtract 9 from 91, because 91 is less than 100 & write square of deviation right side of it.}
  
  82/81
  
  912 &=& 8281
  
  912 &=& 91\pm 9
  
  106^2 \Rightarrow 106
  
  \begin{array}{c}
  \text{106 is 6 more than 100. So deviation is 6, & 106 is more than 100 so add 6 to 106, & write square of 6 right side of it.}
  
  126/13 \begin{array}{c}
  \text{Square of 13 is in 3 digit so add 1 to the left side number.}
  
  114^2 &=& 12769
  
  \begin{array}{c}
  \text{512 is 12 more than 500, add 12 in 250 & write, 12^2 right side of it.}
  
  512^2 \Rightarrow 250
  
  \begin{array}{c}
  124^2 &=& 15376
  
  \text{deviation is 112 & square of deviation is 12544}
  
  \text{Exercise For Practice}

(Try to solve all the questions mentally)

(1) \( 53^2 \)  (2) \( 47^2 \)  (3) \( 93^2 \)  (4) \( 98^2 \)

3. Square of any number near 500

For near 50

\[ N^2 = (25 \pm \text{deviation})^2 \]

\[ \Rightarrow 2\text{-digit} \]

For near 500

\[ N^2 = (250 \pm \text{deviation})^2 \]

\[ \Rightarrow 3\text{-digit} \]

Here base is 250 & we write the square of deviation in 3 digit.

- \[ 512^2 \Rightarrow 250 \]
  \[ \begin{array}{c}
  \text{512 is 12 more than 500, add 12 in 250 & write, 12^2 right side of it.}
  
  512^2 &=& 262144
  
  \begin{array}{c}
  \text{462^2 \Rightarrow 250}
  
  \begin{array}{c}
  \text{deviation is 112 & square of deviation is 12544}
  
  462^2 &=& 213444
  
  \begin{array}{c}
  \text{Exercise For Practice}

(Try to solve all the questions mentally)

(1) \( 53^2 \)  (2) \( 47^2 \)  (3) \( 93^2 \)  (4) \( 98^2 \)

4. Square of any number near 1000

For near 100

\[ N^2 = (N \pm \text{deviation})^2 \]

\[ \Rightarrow 2\text{-digit} \]

For near 1000

\[ N^2 = (N \pm \text{deviation})^2 \]

\[ \Rightarrow 3\text{-digit} \]
(1014)^2 \Rightarrow 1014 
\begin{align*}
&+ 14 \\
&1028/196 \\
1014^2 &= 1028196 \\
\end{align*}

(984)^2 \Rightarrow 984 
\begin{align*}
&-16 \\
&968/256 \\
984^2 &= 968256 \\
\end{align*}

(1056)^2 \Rightarrow 1056 
\begin{align*}
&+ 56 \\
&1112 / 3136 \\
1056^2 &= 1115136 \\
\end{align*}

(884)^2 \Rightarrow 884 
\begin{align*}
&-116 \\
&768/13456 \\
884^2 &= 781456 \\
\end{align*}

Exercise For Practice
(Try to calculate all the question mentally)

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<td>(393)^2</td>
<td>(444)^2</td>
<td>(532)^2</td>
</tr>
<tr>
<td>(612)^2</td>
<td>(576)^2</td>
<td>(1112)^2</td>
</tr>
<tr>
<td>(991)^2</td>
<td>(942)^2</td>
<td>(1011)^2</td>
</tr>
<tr>
<td>(506)^2</td>
<td>(1015)^2</td>
<td>(1036)^2</td>
</tr>
<tr>
<td>(486)^2</td>
<td>(598)^2</td>
<td>(582)^2</td>
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ANSWERS

| (1) 154449 | (2) 197136 | (3) 283024 |
| (4) 374544 | (5) 331776 | (6) 1236544 |
| (7) 982081 | (8) 887364 | (9) 1022121 |
| (10) 256036 | (11) 1030225 | (12) 1073296 |
| (13) 326196 | (14) 357604 | (15) 338724 |

5 Square of any number near 200
Let us have to find the square of 232. We can find the square of 232 by two way as a rule of near 100 & as a rule of near 50.

<table>
<thead>
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<th>By the rule of near 50</th>
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<td>200 is two times of 100 &amp; 232 is 32 more than 200. We use the following formula For near 200</td>
<td>200 is 4 times of 50, take base 400 for it, as we have discussed earlier &amp; we can use the following formula for near 200.</td>
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<tr>
<td>( N^2=2(N \pm d)/(\text{deviation})^2 )</td>
<td>( N^2 = (400 \pm 4)/(\text{deviation})^2 )</td>
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<tr>
<td>((232)^2 = 2(232 + 32)/(32)^2)</td>
<td>((232)^2 = (400 + 4 \times 32)/(32)^2)</td>
</tr>
<tr>
<td>( = 528 / 1024 = 53824 )</td>
<td>( = 528 / 1024 = 53824 )</td>
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\[
\text{We can use any one of them which is comfortable for us. Normally rule of near 50 is more comfortable then rule of near 100.}
\]

\[
\begin{align*}
(214)^2 &= (400 + 4 \times 14 + (14)^2) \\
&= 456 / 196 = 45796 \\
(196)^2 &= (400 - 4 \times 4) / 4^2 = 38416
\end{align*}
\]

6. Square of any number near 300
Take base 900 & use the following formula

\[
N^2 = (900 \pm 6 \times \text{deviation}) \times (\text{deviation})^2
\]

\[
300 \text{ is 6 times of 50} \\
(306)^2 = (900 + 6 \times 6) / 6 = 93636 \\
(321)^2 = (900 + 6 \times 21) / (21)^2 = 1026 / 441 = 103041 \\
(289)^2 = (900 - 6 \times 11) / 11^2 = 834 / 121 = 83521
\]

Application of squares

1. Multiplication of two even or odd numbers
In this method we have to find the middle no. of two numbers which we have to multiply. For example middle number of 24 & 28 is 26. For middle number take half of the difference of the numbers. Difference between 24 & 28 is 4, half of it 2. So middle number of 24 & 28 is 2 more then 24 or 2 less than 28.

Middle no. of 35 & 59, difference between 35 & 59 is 24 So, middle no. is 12 more than 36 or 12 less than 59 which is 47.

We can use the following formula for finding the multiplication of two even or odd numbers.

\[
(\text{middle})^2 - (\text{common difference of middle no. from the number})^2
\]

Proof

\[
\begin{align*}
63 \times 57 &\Rightarrow (60 + 3)(60 - 3) \text{ (middle number is 60)} \\
&\Rightarrow 60^2 - 3^2 \Rightarrow 3600 - 9 = 3581 \\
58 \times 64 &\Rightarrow 61^2 - 3^2 \text{ (middle no. is 61)} \\
&\Rightarrow 3721 - 9 = 3712 \\
62 \times 44 &\Rightarrow 50^2 - 9^2 = 2809 - 81 = 2728 \\
103 \times 77 &\Rightarrow 90^2 - 13^2 = 8100 - 169 = 7931
\end{align*}
\]
(2) Product of two consecutive numbers

\[ \text{Product of two consecutive numbers} = (\text{smaller no.})^2 + (\text{smaller no.}) \]

(i) \[ 62 \times 63 = (62)^2 + 62 = 3844 + 62 = 3906 \]

(ii) \[ 73 \times 74 = (73)^2 + 73 = 5329 + 73 = 5402 \]

(iii) \[ 512 \times 513 = 262144 + 512 = 262656 \]

(iv) \[ 1048 \times 1049 = 1098304 + 1048 = 1099352 \]

Another method for finding the squares of any number

\[ (a b)^2 = a^2 + b^2 \]

1 digit

Where, \( b = \text{unit place, } a = \text{remaining number} \)

- \(32)^2 = 3^2 \times 2 \times 2 = 124\]

\( a \cdot b = 9 \div 124 \)

Since middle part is in 2 digit. So transfer 1 to left side.

\[ = 1024 \]

- \(53)^2 = 5^2 \times 3 \times 9 = 25 \div 90 = 2809 \]

- \(153)^2 = 225 \div 90 = 23409 \]

- \(42)^2 = 1764 \]

- \(612)^2 = 3721 \div 4 = 3744 \]

SOLVED EXAMPLES

Ex.1 \[ \frac{(3+3+3+3)}{5+5+5+5+5} = ? \]

Sol. \[ \frac{(3+3+3+3)}{5+5+5+5+5} = ? \]

\[ \Rightarrow \frac{12}{20} = ? \]

\[ \Rightarrow \frac{4}{5} = ? \]

\[ \Rightarrow ? = 1 \]

Ex.2 \[ \frac{5 + 2 \times 19 - 15 - 7}{3 \times 13 - 156} = 6 \]

Sol. \[ \frac{5 + ? \times 19 - 15 - 7}{3 \times 13 - 156} = 6 \]

\[ \Rightarrow ? \times 19 - 17 = 169 - 156 \]

\[ \Rightarrow 19 - 17 = 13 \times 6 \]

Ex.3 \[ 35 - [23 - (19 \cdot (15 - x))] = 12 \times 8 + \frac{1}{2} \]

Sol. \[ 35 - [23 - (19 \cdot (15 - x))] \]

\[ \Rightarrow 35 - 23 + (19 - 15 + x) = 48 \]

\[ \Rightarrow 12 + 4 + x = 48 \]

\[ \Rightarrow x = 48 - (4+12) = 32 \]

Ex.4 \[ 261 + ? \times 15 + 270 = 405 \]

Sol. \[ 261 + ? \times 15 + 270 = 405 \]

\[ 261 + ? \times 15 = 405 - 270 \]

\[ \Rightarrow \frac{261 \times 15}{135} = ? \]

\[ \Rightarrow ? = 29 \]

Ex.5 \[ \frac{1 + 5 + \frac{1}{4} + 2 + \frac{1}{4}}{3} = ? \]

Sol. \[ \frac{1 + 5 + \frac{1}{4} + 2 + \frac{1}{4}}{3} = \frac{11}{3} \]

\[ ? = 11 - \left( \frac{3 + 5 + 2}{5} \right) \]

\[ = \frac{34}{3} \left( \frac{7 + 26 + 9}{4} \right) \]

\[ = \frac{34}{3} \left( \frac{219}{4} \right) = \frac{23}{20} \]

\[ = 60 \]
EXERCISE

1. If \( a + \frac{1}{a} = 4 \) then the value of \( a^2 + \frac{1}{a^2} \)
   (a) 14  (b) 16  (c) 18  (d) None

2. If \( x^2 + 1/x^2 = 6 \) then \( x - 1/x = \)
   (a) 0  (b) \pm 2  (c) \pm 1  (d) None

3. If \( x/y + y/x = 10 \) then \( x^2/y^2 + y^2/x^2 = \)
   (a) 100  (b) 98  (c) 102  (d) None

4. \( (101)^2 - (100)^2 = \)
   (a) 201  (b) 102  (c) 10100  (d) None

5. Find the value of \( \frac{25 \times 25 + 13 \times 13 - 2 \times 25 \times 13}{12} \)
   (a) 1  (b) 12  (c) 38  (d) None

6. \( \frac{a^3 + b^3}{a^2 - ab + b^2} = \)
   (a) \( a + b \)  (b) \( a - b \)  (c) \( a^2 + b^2 \)  (d) None

7. \( \frac{0.125 + 0.027}{0.25 - 0.15 + 0.09} = \)
   (a) 0.6  (b) 0.2  (c) 0.8  (d) 0.14

8. \( 103 \times 97 = \)
   (a) 9991  (b) 9999  (c) 10397  (d) None

9. If \( a + b + c = 0 \) then the value of \( a^2/bc + b^2/ca + c^2/ab \)
   (a) 1  (b) \( abc \)  (c) 3  (d) abc

10. If \( 8.1 \times 8.1 - 1.9 \times 1.9 = k \) then the value of \( k \)
   (a) 26  (b) 10  (c) 62  (d) None

11. \( \frac{(0.03)^2 - (0.01)^2}{(0.03 - 0.01)} = \)
    (a) 0.02  (b) 0.004  (c) 0.4  (d) 0.04

12. \( (5.5)^3 - (4.5)^3 = \)
    (a) 1  (b) 75  (c) 74.25  (d) 75.25

13. \( 6.5 \times 6.5 - 45.5 + 3.5 \times 3.5 \) then the value of \( k \)
    (a) 10  (b) 9  (c) 5  (d) 3

14. \( \frac{(149 \times 149) - (5.1 \times 5.1)}{149 - 5.1} = \)
    (a) 0.20  (b) 20.00  (c) 2  (d) None

15. The value of \( (0.98)^2 + (0.02)^2 + 3 \times 0.98 \times 0.02 - 1 \)
    (a) 1.98  (b) 1.09  (c) 1  (d) 0

16. \( \frac{(598 + 178)^2 - (598 - 178)^2}{598 \times 178} = ? \)
    (a) 1/2  (b) 4  (c) 402  (d) 209/388

17. \( 10010.001 + 1001.01 + 101.1 = ? \)
    (a) 12121.011  (b) 11111.11  (c) 122203.101  (d) 11203.111

18. \( x^2 + 1/x^2 = ? \)
    (a) \( x + 1/x \)  (b) \( x - 1/x \)  (c) \( x + 1/x \)  (d) (a) and (b) both

19. \( x^2 = 4 \) and \( y^2 = 9 \) then \( (x - y)^2 = \)
    (a) 1  (b) 13  (c) 5  (d) None

20. \( (x + 1/x)^2 = \)
    (a) \( x - 1/x \)  (b) \( x + 1/x \)  (c) \( x - 1/x \)  (d) None

21. \( 4^2 \times \frac{9}{14} + 5 \times \frac{2}{5} = \)
    (a) 15  (b) \( \frac{2}{5} \)  (c) \( 14 \frac{2}{5} \)  (d) \( 14 \frac{4}{5} \)

22. \( 2 + \frac{1}{2 + \frac{1}{2}} = \)
    (a) 37/12  (b) 27/5  (c) 12/29  (d) 29/12

23. \( 0.2 \times 0.2 + 0.02 \times 0.02 - 0.4 \times 0.02 = \)
    (a) 0.09  (b) 0.9  (c) 0.009  (d) 9

24. \( \frac{1}{1 + \frac{1}{1 + \frac{1}{3}}} = \)
    (a) 1  (b) 3  (c) 1/3  (d) 1\frac{1}{3}

25. \( (x + 1/x)^3 - 3(x + 1/x) = \)
    (a) \( x^3 + 1/x^3 \)  (b) \( x^3 - 1/x^3 \)  (c) \( a \) and \( b \) both  (d) None

26. \( (a + b)^2 - (a - b)^2 = \)
    (a) 4ab  (b) 2(a^2 + b^2)  (c) 2(a^2 - b^2)  (d) None
27. \[ 1 + \frac{2}{3} \div \frac{1 + \frac{1}{2}}{1 + \frac{1}{2}} = \]
(a) 13/21  (b) 21/13  (c) 13/4  (d) 4/13

28. \[ \frac{(x/y) + (y/x)}{2} - \left( \frac{x/y}{2} - \left( \frac{y/x}{2} \right) \right) = \]
(a) 2  (b) \(2[(x/y)^2 + (y/x)^2]\)  (c) \(x/(y^2) - (y/x)^2\)  (d) None

29. \[ \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{ab + bc + ca}{x} \text{ them } x = \]
(a) abc  (b) \(a + b + c\)  (c) \(\frac{n+b+c}{abc}\)  (d) None

30. \[ 1032.64 + 456.79 - 282.16 = ? \]
(a) 1207.27  (b) 545.64  (c) 585.94  (d) 572.94

31. \[ \sqrt{(12)^3} = ? \]
(a) 12  (b) 144  (c) 20736  (d) 20636

32. \[ \left( (24)^2 + (36)^2 \right) / 25 = ? \]
(a) 70.88  (b) 74.88  (c) 72.78  (d) 74.78

33. \[ (144)^2 + (12)^2 = ? \]
(a) 8  (b) 144  (c) 24  (d) 120

34. \[ \frac{13}{32} + \frac{3}{4} - \frac{11}{14} = ? \]
(a) \(\frac{2}{224}\)  (b) \(\frac{3}{224}\)  (c) \(\frac{81}{224}\)  (d) None.

35. \[ (?)^3 = 15625 \]
(a) 25  (b) 40  (c) 80  (d) 2

36. \[ \sqrt[3]{551368} = ? \]
(a) 78  (b) 82  (c) 72  (d) 84

37. \[ \sqrt[3]{2704} = ? \]
(a) 62  (b) 13  (c) 26  (d) 52

38. \[ \sqrt[3]{10201 - 3136} = ? \]
(a) 45  (b) 40  (c) 35  (d) 30

39. \[ -30 - 45 - 31 + 62 + 2 \times 10 = ? \]
(a) -52  (b) -105  (c) -138  (d) -85

40. \[ \sqrt{5249 + (75)^2 + \sqrt{5} = 5745} \]
(a) 3721  (b) 4096  (c) 3481  (d) 3969

41. \[ 42\% \text{ of } 386 + ? = 165 \]
(a) 221.13  (b) 225.25  (c) 211.13  (d) None

42. \[ 7^2 + \{8^2 - (2^2 + 5^2)\} = ? \]
(a) 352  (b) 350  (c) 359  (d) 378

43. \[ 48096 + \sqrt{2} = 167 \times 18 \]
(a) 31  (b) 32  (c) 256  (d) 961

44. \[ \frac{586 \times 8 + 12}{5^3 - (6^2 + 19)} = ? \]
(a) 130  (b) 2  (c) 6  (d) 3

45. \[ 1032.64 + 456.79 - 282.16 = ? \]
(a) 1207.27  (b) 545.64  (c) 585.94  (d) 572.94

NOTES:
2 NUMBER SYSTEM

Numbers can be classified in various categories as follows

Numbers

Real Number

Imaginary Number

Rational Number

Irrational Number

Integer

Fraction

Negative Integer

Whole Number

Proper Improper Mixed

Zero

Positive Integer

(Natural Number)

Rational numbers can be both positive or negative.
e.g. \( \frac{3}{1}, \frac{5}{2}, \frac{6}{7}, \frac{3}{4}, \ldots \) etc.

Every rational number when expressed in decimal form is expressible in either terminating decimals or repeating decimals.

e.g. \( \frac{4}{9} = 0.444444444 \ldots \ldots \frac{1}{5} = 0.2 \)

Irrational numbers

Numbers those when expressed in decimal form are neither terminating nor repeating are known as irrational numbers.
e.g. \( \sqrt{3}, \sqrt{7}, \pi \) etc.

Integers

An integer is any number of the set
\( \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \} \)

Corresponding to every natural number, there is a 'negative' number. These negative numbers and natural numbers form the set of integers along with 0. A set of integers is denoted by 'I'.

The order of relation in the set I is \( \ldots < -3 < -2 < -1 < 0 < 1 < 2 < 3 < \ldots \)

Natural numbers

All the non negative counting numbers are known as natural numbers.
e.g. 1, 2, 3, 4, 5,........ is a set of natural numbers. 0 is not included in the set of natural numbers.

Whole numbers

All the Natural numbers including 0 forms the set known as whole numbers.
e.g. 0, 1, 2, 3, 4, 5 ........ is a set of whole numbers.

Here, we can say that all natural numbers are also whole numbers.

Even numbers

All integers that are divisible by 2 are known as even numbers.
e.g. - 2, -4, 12, 16 ..... etc are even numbers.

Odd numbers

All integers that are not divisible by 2 are known as odd numbers.
e.g. 3,1,-5,7,-11 etc are odd numbers.
DIVISIBILITY RULES

1. Divisibility by 2
A number is exactly divisible by 2, if the last digit of that number is 0, 2, 4, 6 or 8.
- e.g. 16, 18, 24, 36, 42, etc. are exactly divisible by 2.
\[ 4 + 6 + 2 = 12 \]
\[ 12 \text{ is divisible by } 3. \]
Hence, 462 is divisible by 3.

2. Divisibility by 3
A number is exactly divisible by 3, if the sum of all its digit is divisible by 3.
- e.g. 462
\[ 4 + 6 + 2 = 12 \]
\[ 12 \text{ is divisible by } 3. \]
Hence, 462 is divisible by 3.

3. Divisibility by 4
A number is exactly divisible by 4, if the last two digits of that number are either 00 or divisible by 4.
- e.g. 1728
Here, last two digits are 28. 28 is divisible by 4.
Hence, 1728 is divisible by 4.

4. Divisibility by 5
A number is exactly divisible by 5, if its last digit is either 0 or 5.

5. Divisibility by 6
A number is exactly divisible by 6, if it is divisible by 2 as well as 3.
- e.g. 54 is divisible by 2 as well as by 3.
Hence, 54 is divisible by 6.

6. Divisibility by 8
A number is exactly divisible by 8; if the number formed by its last three digits of that number are either 000 divisible by 8.
- e.g. 12520
Here, last three digits are 520. 520 is divisible by 8.
Hence, 12520 is divisible by 8.

7. Divisibility by 11
A number is exactly divisible by 11, if the difference between the sum of digits at odd places and the sum of digits at even places is either 0 or divisible by 11.
- e.g. 121
\[ \text{Sum of digits at odd places} = 1 + 1 = 2 \]
\[ \text{Sum of digits at even places} = 2 \]
\[ 2 - 2 = 0 \therefore 121 \text{ is divisible by } 11. \]
- e.g. 563574
\[ \text{Sum of digits at odd places} = 5 + 3 + 7 = 15 \]
\[ \text{Sum of digits at even places} = 4 + 5 + 6 = 15 \]
\[ 15 - 15 = 0 \therefore 563574 \text{ is divisible by } 11. \]
Factor and multiples

A number is a factor of another number if it divides that number exactly i.e. A is a factor of B if A exactly divides B.

- e.g. 30 is exactly divisible by 5.
- Hence, 5 is a factor of 30.

If A is a factor of B, then B is called a multiple of A.

Hence, 30 is multiple of 5.

SOME IMPORTANT FORMULAE

- \((a + b)^2 = a^2 + b^2 + 2ab\)
- \((a - b)^2 = a^2 + b^2 - 2ab\)
- \((a + b)^2 - (a - b)^2 = 4ab\)
- \((a + b)^2 + (a - b)^2 = 2(a^2 + b^2)\)
- \((a^2 - b^2) = (a + b)(a - b)\)
- \((a^3 + b^3) = (a + b)(a^2 - ab + b^2)\)
- \((a^3 - b^3) = (a - b)(a^2 + ab + b^2)\)
- \(a^3 + b^3 + c^3 - 3abc = (a + b + c)\)
- \(x(a^3 + b^3 + c^3 - 3abc = 0)\) ; then either \(a + b + c = 0\).

SOLVED EXAMPLES

Ex. 1. Express \(0.\overline{345}\) as a fraction.

\[\Rightarrow \quad \text{Let } x = 0.\overline{345} \Rightarrow 1000x = 345.\overline{345} \cdot \overline{345} \cdot \overline{345} \cdot 0.345 \Rightarrow 999x = 345 \Rightarrow x = \frac{345}{999}\]

Ex. 2. Find the total number of factors of 462.

\[\Rightarrow \quad \text{The factorized form of 462 is } (2 \times 3 \times 7 \times 11)\]

So the total number of factors is \((1 + 1)(1 + 1)(1 + 1)(1 + 1) = 2 \times 2 \times 2 \times 2 = 16\)

Ex. 3. What is remainder if \(7^{23}\) is divided by 6?

\[\frac{7^{23}}{6} = 7 \times 7 \times 7 \ldots \ldots (23 \text{ times}) \Rightarrow \frac{6}{6} \Rightarrow 7 \text{ divided by 6 leaves remainder 1.} \]

Thus, \(7^{23}\) when divided by 6 will leave remainder \(1 \times 1 \times 1 \ldots \ldots (23 \text{ times}) = 1\)

Ex. 4. Evaluate \((52)^2 + (48)^2 + 2 \times 52 \times 48\)

\[\Rightarrow \quad (a^2 + b^2 + 2ab) = (a + b)^2 \]

\((52 + 48)^2 = 100^2 = 10000\)

Ex. 5. Find the unit digit in the product \(269 \times 541 \times 366 \times 345 \times 38\)

\[\Rightarrow \quad \text{Unit digit in the product } = 9 \times 1 \times 6 \times 5 \times 8 = 0\]

EXERCISE

1. Find the unit digit of \((23758)^{149}\).
   (a) 2  (b) 4  (c) 6  (d) 8

2. If a number is represented by N = 84\times192217301. What will be remainder when N is divided by 27?
   (a) 7  (b) 6  (c) 5  (d) 9

3. The remainder when \(6^{100}\) is divided by 7, is : 
   (a) 3  (b) 0  (c) 1  (d) 2

4. If \(10 \frac{1}{3} + 15 \frac{1}{3} + x = \frac{404}{3}\) Find the value of x.
   (a) \(\frac{12}{15}\)  (b) \(\frac{11}{17}\)  (c) \(\frac{2}{3}\)  (d) 17

5. \(85 + 5 + (5 - 9 \times 6) + 7\)
   (a) 2  (b) 0  (c) 24  (d) 17

6. The value of \(2,427\) in the form of fraction is 
   (a) \(\frac{136}{330}\)  (b) \(\frac{73}{280}\)  (c) \(\frac{77}{180}\)  (d) \(\frac{161}{3}\)

7. Two no. are such that their sum is 11 and their difference is 7. Find their product:
   (a) 8  (b) 30  (c) 18  (d) None

8. The value of \(\sqrt[3]{3\sqrt{3\sqrt{3\ldots\infty}}}\) is 
   (a) 0  (b) 1  (c) \(3\sqrt{3}\)  (d) 3

9. If \(a = 7 - \frac{10}{a}\) then find a:
   (a) 10, -2  (b) 2, 5  (c) -2, 5  (d) None

10. The value of the expression \(\left[1 - \frac{1}{2}\right] \left[1 - \frac{1}{3}\right] \ldots \left[1 - \frac{1}{n}\right]\) is equal to:
    (a) \(\frac{1}{n}\)  (b) \(\frac{2}{n}\)  (c) \(\frac{n(n-1)}{n}\)  (d) \(\frac{n}{n}\)

11. \(\frac{167 \times 167 + 167 \times 160 + 160 \times 160}{167 \times 167 - 167 \times 160 - 160 \times 160}\) is equal to
    (a) 7  (b) \(\frac{1}{7}\)  (c) \(\frac{1}{327}\)  (d) 327
12. The unit digit in the product of $3217^{93}$ is:
   (a) 3     (b) 7     (c) 9     (d) 1

13. For any natural number $n$, $n^2 + n^2$ is always
   (a) odd     (b) even     (c) either even or odd     (d) Can't say

14. A number when divided by 117 leaves 17 as remainder.
   If the same number is divided by 13, the remainder obtained is:
   (a) 2     (b) 1     (c) 11     (d) 4

15. By what smallest number, 32500 must be multiplied or divided in order to make it a perfect square?
   (a) 6     (b) 5     (c) 13     (d) 10

16. What should come in place of the question mark in the following equation?
   $\% \text{ of } 6110 = 47 \times 52$
   (a) 30     (b) 40     (c) 50     (d) 60

17. Find the value
   \[ \frac{7}{9} \times \frac{2}{3} \times \frac{28}{5} \times \frac{40}{3} = \]
   (a) 76     (b) 77\frac{13}{21}     (c) 76\frac{13}{21}     (d) 77\frac{2}{7}

18. Find the value of
   \[ (15.5)^2 + (8)^2 + (13)^2 - (28)^2 \]
   (a) 132.75     (b) 135.25     (c) 137.25     (d) 137.75

19. If $x = \sqrt{6} + \sqrt{6} + \sqrt{6} + \ldots$, then what would be the value of $x$?
   (a) 2     (b) 3     (c) 4     (d) 5

20. What would be the value of $y$, if $\frac{1}{x} + \frac{2}{y} = \frac{3}{z}$?
   (a) $\frac{xy}{2z-x}$     (b) $\frac{2zx}{3y-x}$     (c) $\frac{2zn}{3x-z}$     (d) $z$

21. Find the value of
   \[ \frac{1}{1 \times 2} - \frac{1}{2 \times 3} + \frac{1}{3 \times 4} \]
   (a) $x$     (b) $1/x$     (c) 0     (d) None

22. \[ \frac{743 + 343^2 + (743 - 343)^2}{743 \times 743 + 343 \times 343} \]
   is equal to:
   (a) 1     (b) 719     (c) 2     (d) 965

23. The last digit of the number $1^2 + 2^2 + 3^2 + \ldots + 99^2$ is
   (a) 0     (b) 1     (c) 2     (d) 4

24. Which among the following is greatest $\sqrt{6}$, $\sqrt{12}$, $\sqrt{18}$, $\sqrt{26}$?
   (a) $\sqrt{6}$     (b) $\sqrt{12}$     (c) $\sqrt{18}$     (d) All are equal

25. Which among the following is greatest
   $\sqrt{5} + \sqrt{5}$, $\sqrt{6} + \sqrt{5}$, $\sqrt{7} + \sqrt{4}$
   (a) $\sqrt{5} + \sqrt{5}$     (b) $\sqrt{6} + \sqrt{5}$     (c) $\sqrt{7} + \sqrt{4}$     (d) All are equal

26. The unit digit in the expression
   \[ (31^1 + 32^2 + 33^3 + 34^4 + 35^5 + 36^6) \]
   (a) 1     (b) 9     (c) 7     (d) 0

27. Simplify
   \[ \frac{(7.64 \times 7.64 \times 7.64) + (8.36 \times 8.36 \times 8.36)}{(7.64 \times 7.64) - (8.36 \times 8.36)} + (8.36 \times 8.36) \]
   (a) 0     (b) 1     (c) 16     (d) 13

28. How many numbers between 300 and 600 are divisible by 4, 5, and 6?
   (a) 5     (b) 6     (c) 4     (d) 7

29. If $\sqrt[6]{x + \sqrt[6]{x + \ldots}}$, then what would be the value of $x$?
   (a) 2     (b) 3     (c) 4     (d) 5

30. What would be the value of $y$, if $\frac{1}{x} + \frac{2}{y} = \frac{3}{z}$?
   (a) $\frac{xy}{2z-x}$     (b) $\frac{2zx}{3y-x}$     (c) $\frac{2zn}{3x-z}$     (d) $z$

31. Find the remainder when $((111)^{11})^{11}$ is divided by 9.
   (a) 8     (b) 9     (c) 2     (d) 1

32. $25 \sqrt[7]{49 \sqrt[5]{7} + 1}$: Find $x$ and $y$.
   (a) 2, $\frac{3}{2}$     (b) $\frac{3}{2}$, 2     (c) 3, 2     (d) 2, 3

33. A group of players play a game in which each player contributes an amount equal to the number of players in the group and the winner of the game gets this entire amount as the prize money. The number of times they play the game is equal to the number of players in their group. If a person who wins half the number of games earns four times the money he contributes for each game, then the number of players in the group is:
   (a) 2     (b) 4     (c) 16     (d) 8
34. A computer program when fed with a real number \( x \), generates another number \( y \) such that \( y \) is the greatest integer less than or equal to \( x \). Amisha inputs the values \( \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4} \ldots \ldots \sqrt{100} \) sequentially and finds the product of all the numbers that the computer gives as output. The product is:
   (a) \( 2^{32} \times 3^{10} \times 5^{11} \times 7^{11} \) 
   (b) \( 2^{36} \times 3^{10} \times 5^{11} \times 7^{11} \) 
   (c) \( 2^{36} \times 3^{10} \times 5^{11} \times 7^{11} \) 
   (d) \( 2^{32} \times 3^{10} \times 5^{11} \times 7^{11} \)

35. \((100)! \) in base 10 when written in base 14 will end with 16 zeroes. \((200)! \) in base 10 when written in base 34 will end with how many zeroes?
   (a) 49 
   (b) 11 
   (c) 52 
   (d) 26

36. If \( 0 < x < 100 \), find the number of integers such that \( x \) is equal to the product of 3 and an integer.
   (a) 32 
   (b) 33 
   (c) 37 
   (d) 85

37. How many even integers \( n \), where \( 100 \leq n \leq 200 \), are divisible neither by seven nor by nine?
   (a) 36 
   (b) 39 
   (c) 34 
   (d) 38

38. For a positive integer \( n \), let \( P_n \) denote the product of the digits of \( n \), and \( S_n \) denote the sum of the digits of \( n \). The number of integers between 10 and 1000 for which \( P_n = S_n \) is:
   (a) 81 
   (b) 9 
   (c) 28 
   (d) 19

39. The number of solutions of the equation \( 2x + y = 40 \) where both \( x \) and \( y \) are positive integers and \( x \leq y \) is:
   (a) 7 
   (b) 13 
   (c) 15 
   (d) 28

40. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, \ldots, 9 such that the first digit of the code is nonzero. The code, handwritten on a slip, can however potentially create confusion when read upside down – for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?
   (a) 80 
   (b) 79 
   (c) 71 
   (d) 65

41. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers?
   (a) 26 
   (b) 53 
   (c) 41 
   (d) 67

42. Consider the set \( S = \{1, 2, 3 \ldots 1000\} \). How many arithmetic progressions can be formed from the elements of \( S \) that start with 1 and end with 1000 and have at least 3 elements?
   (a) 3 
   (b) 7 
   (c) 8 
   (d) 5

43. Three boys stole certain number of mangoes and hide them in a safe place. One of the boys secretly ate one mango and stole one third of the remaining ones and went off. After some time the second boy came and did the same thing. Still later the third boy came and repeated the process. On the next day they found that the remaining mangoes could be equally divided among the three of them. How many mangoes did the three boys steal initially?
   (a) 25 
   (b) 31 
   (c) 16 
   (d) None

**Directions (44 - 45)**: Follow the given instructions and choose the correct alternative.

1) \( A \times B = A + B \) if \(|A + B| \) is prime
   = \( A \# B \) otherwise

2) \( A \# B = A - B \) if \(|A - B| \) is prime = 0 otherwise

44. For which of the following conditions, \( (A \times B) = (A \# B) \) does not necessarily hold true?
   (a) \(|A + B| \) is composite
   (b) \( A \) is prime and \( B = 0 \)
   (c) Both \( A \) and \( B \) are prime numbers
   (d) none of the above

45. What is the condition for \( A \# B \) to be always positive?
   (a) \( A > B \) 
   (b) \(|A - B| \) is prime
   (c) \(|A| \leq |B| \) 
   (d) (a) and (b)

**NOTES:**
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